A Method for the Design of Fixed External Sun-Shades

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A procedure for the design of fixed external sun-shades for windows is presented. The window is assumed to be located in a building with a given orientation anywhere in the world. The method is based on the calculation of the length of a pole at any location on the window and able to cast a sufficiently long shadow during the desired period and at the desired hours.

The application of a computer for calculations and a plotter for presenting the results graphically makes the design of sun-shades by means of this method extremely efficient, fast and easy.

An example of the application of this method in the design of sun-shades for windows in an office building is given. The method does not lead to a unique final solution and hence the freedom of the architect is not impaired.

Introduction

The design of sun-shades for windows is very important in countries with hot climates. The role of windows is to permit natural illumination as well as connection with the outside world.[1] One finds, however, in tropical countries, bright skies and a high amount of scattered light during the day almost the whole year around. Consequently, a central problem in the design of the openings in a building is to prevent the penetration into it of direct solar radiation. There is always enough indirect light. The question of illumination —and hence the number of openings and their sizes—is of secondary importance. The penetration of direct sunlight into a building has two effects: (a) to heat it and (b) the creation of strong contrast and glare which disturb visibility.

The energy content in solar radiation is very often neglected. The solar radiation at the top of the atmosphere amounts to 1·94 calories/min/cm^2, known as the solar constant.[2] The attenuation of the radiation in a clear atmosphere depends only on the height of the

Build International (8) (1975)—

sun above the horizon. On a typical summer day, when the sun is about 45° above the horizon, the power deposited by the sun directly through a window with linear dimensions of 2 × 1 m is about 1 kW. This power constitutes a load on any air-conditioning system. Actually, this is the power of typical air-conditioning wall units. (In this calculation, only the additional heat is taken into consideration. Other sources of heat are warm walls and heat produced in the building.) Artificial illumination, on the other hand, consumes much less energy, in particular if fluorescent lamps are used. Moreover, if sun-shades are carefully designed they will prevent the penetration of direct sunlight but allow illumination from the bright sky and surroundings. Scattered light usually has longer wavelengths than direct solar radiation and its ability to penetrate glass is less.

Besides being easily controlled without excessive energy consumption, the illumination problem depends considerably on the conditions inside the building, *i.e.* space dimensions, reflection properties of the materials used, the particular jobs performed, etc. These questions are not discussed in this paper.

The design of sun-shades must go along with the design of the heating system. The reason is that heating by winter sun cannot be neglected. When winter sun is allowed to enter a building the heating system must be flexible enough to take into account the fact that several rooms (but not all) may be overheated. We have tried to demonstrate that the use of sun-shades can save energy. The question arises: Why leave any openings in the building? This question has already been posed by several authors.[1,3,4] Here we emphasise that even if the scattered light penetrating through the window is not sufficient for natural illumination, the additional artificial light will consume less energy. In addition, completely closed spaces present problems of ventilation which again consumes energy. Detailed discussions of additional factors are given by Hopkinson and Kay,[1] Lynes[3] and Givoni *et al.*[4]

The discussion in this work is limited to fixed external sun-shades. These shades prevent the penetration of solar radiation through the glass and hence do not give rise to a 'greenhouse' effect. Sun-shades have a strong impact on the architecture of the building. The appearance of large buildings with many windows and entrances is strongly affected by the shape of the sun-shades and, in many cases, they determine the character of the building. Consequently, as well as being effective in precluding the penetration of direct sunlight, sun-shades should be architecturally pleasing.

In this work a procedure for the determination of the shape of sun-shades, the necessary frequency, etc. is presented.

A common practice is to assume a shape of a small roof or shade, and check to what extent direct solar light penetration is prevented. We invert the problem. Assuming a given window in a wall with prescribed orientation, we find, for each point in the window, the necessary height of a shade sufficient to prevent penetration of direct solar light on given days and at given hours. The results are presented graphically in a form ready to use. The design of the sun-shade itself is not unique and is left to the designer, who is now free to evaluate the shade aesthetically. Several examples are given.

Determination of the shadow

The construction of the shadow vector at a given time is based on the position of the sun at that particular moment. Consider Fig. 1 in which the observer's celestial hemisphere is shown. The observer is located at

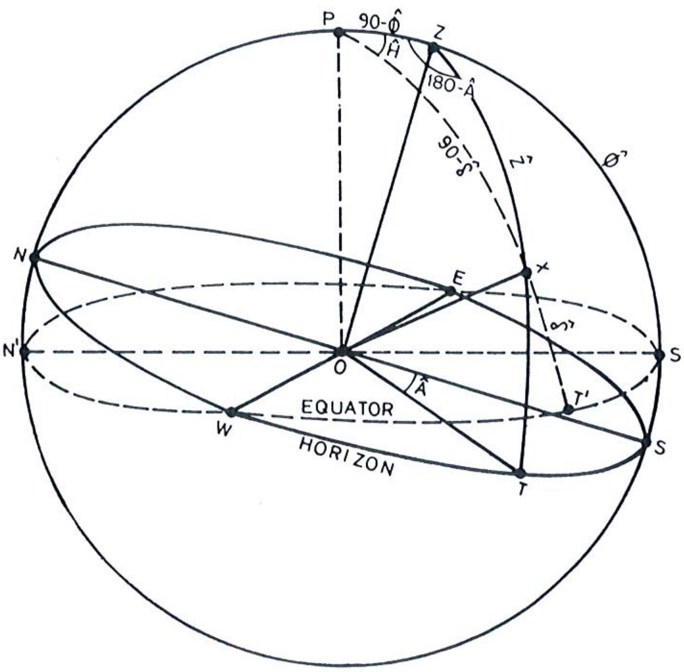

Fig. 1. *The observer's celestial hemisphere.*

point O and his horizon is described by the plane determined by the point $ENWS$. The point Z is the observer's zenith point. Let the observer's north, west, south and east be N, W, S, and E respectively. We assume the observer to use a cartesian system of coordinates with the following fundamental unit vectors: **N, E** and **Z** in the north, east and zenith directions, respectively. Imagine the sun is seen by the observer at point X in the sky. The great circle from the zenith through X meets the horizon at T. The observer describes the position of the sun using two angles: $\hat{Z} = ZOX$—the angle between the zenith and the sun—and $\hat{A} = TOS$—the angle between the sun and the south. The angles $\hat{Z}$ and $\hat{A}$ are the basic data for the calculation of the shadow vector. These angles vary with the date, hour and geographical location of the observer and must be evaluated from fundamental astronomical data.

For the derivation of angles $\hat{Z}$ and $\hat{A}$ we refer to Fig. 1. Let $S'EN'W$ be the projection of the earth's equator on the celestial sphere as seen by the observer at O. (The part $WS'E$ is above the horizon, while $EN'W$ is below it.) Let P be the projection of the North Pole on the celestial sphere as seen by the observer. If $\hat{\Phi}$ is the geographical latitude of the observer, the angle between the projection of the North Pole and the zenith is 90-$\hat{\Phi}$ degrees (Fig. 2). The fundamental given data of the coordinates of the sun are the declination δ of the sun—the angle above or below the equator—and the hour angle, $\hat{H} = ZPX$ (defined as negative in the morning and positive in the afternoon).

Consider the spherical triangle PZX. One finds the following expression for $\hat{Z}$ and $\hat{A}$.

$$\cos \hat{Z} = \sin \hat{\Phi} \sin \delta + \cos \hat{\Phi} \cos \delta \cos \hat{H}, \qquad (1)$$

$$\cos \hat{A} = (\sin \delta - \sin \hat{\Phi} \cos \hat{Z})/\cos \hat{\Phi} \sin \hat{Z}. \qquad (2)$$

The declination of the sun varies from +23° to −23° in six months, while the hour angle varies by 360° in 24 h. Hence, the declination can be considered constant during the day and $\hat{H}$ varies from hour to hour. If k is the hour at which the solar position is wanted, the hour angle $\hat{H} = \hat{H}(k)$ is given by:

$$\hat{H} = \hat{H}(12) - 15(12 - k), \qquad (3)$$

Where $\hat{H}(12)$ is the solar hour angle at the local civilian noon and is calculated directly from the 'Ephemeris Transit Time' which, for our purposes here, is practically equal to the Universal Time at which the sun crosses the Greenwich meridian.

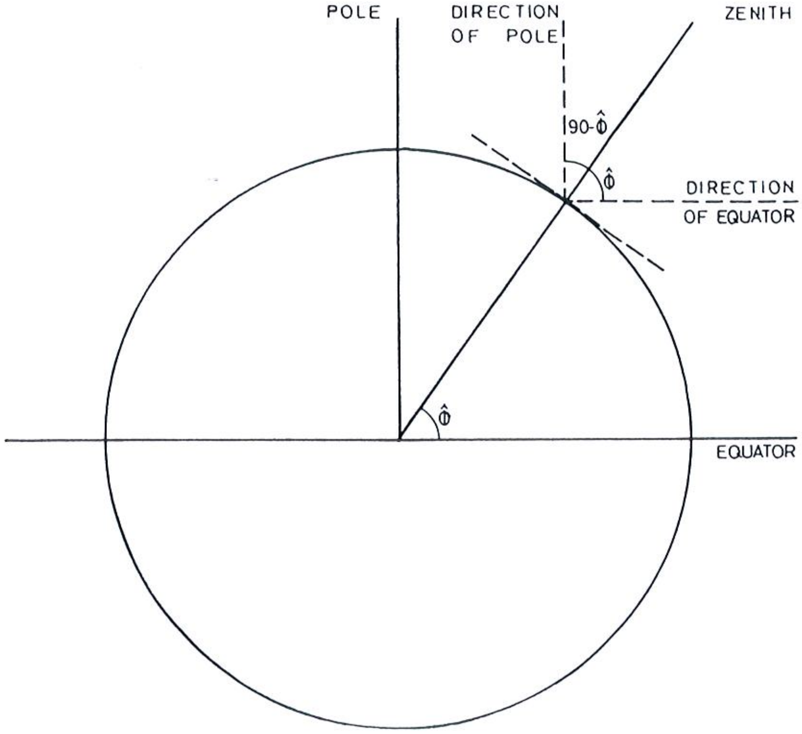

Fig. 2. *The projection of the equator and north pole on the celestial hemisphere as seen by an observer at latitude* $\hat{\Phi}$.

The astronomical data of the solar declination and transit time for every day in the year was taken from the Nautical Almanac for 1973.[5] Although the data change slightly from one year to the next, the changes over more than a century are negligible for the purposes of the calculation of solar shades. The geographical longitude (which is needed for the evaluation of the hour angle according to the local civilian time) and the observer's latitude, as well as the day and hour, are all input for the calculation of the shadow length and direction. The astronomical data for every day are stored in the computer.

The shadow vector **S** is defined as a unit vector in the direction *XO* (from the sun to the observer, see Fig. 3). Let us calculate the components of **S** in the observer's system of coordinates. The vertical

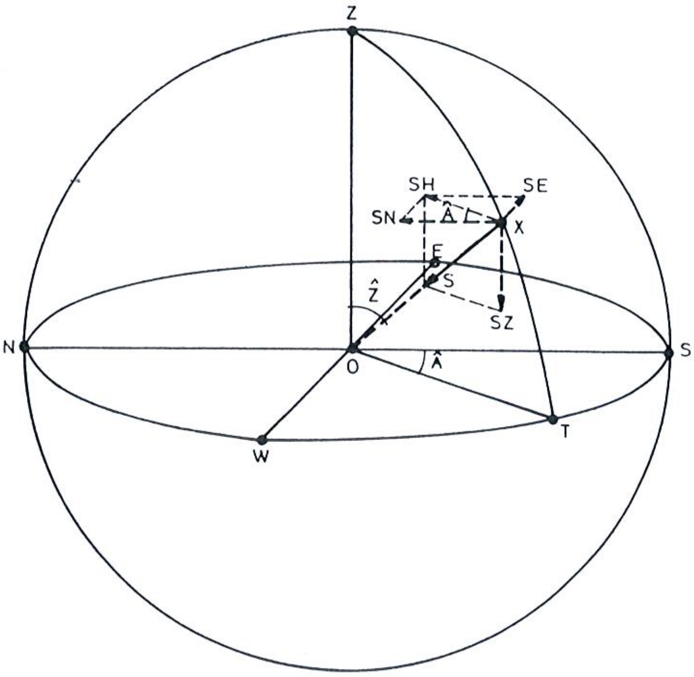

Fig. 3. *The construction of the shadow vector* **S**.

component of **S**, *Sz*, and the in-plane component, *Sh*, are given by:

$$Sz = -\cos \hat{Z}, \qquad Sh = \sin \hat{Z}$$

and hence the north, east and zenith components are:

$$\mathbf{S} = (\cos \hat{A} \sin \hat{Z}, \sin \hat{A} \sin \hat{Z}, -\cos \hat{Z}) \qquad (4)$$

Let **P** be a vector representing a pole of length |**P**| placed at an arbitrary position on the vertical wall (see Fig. 4). We look for the shadow cast by the pole on the wall. The wall is represented by the normal **V**, *i.e.* **V** is a unit vector perpendicular to the wall and in the direction of the outside of the building. The pole **P** can have an arbitrary inclination to the wall. Let **F** be the shadow of the pole **P** on the wall. The shadow vector **F** is determined by the following conditions.

(I) **F** is embedded in the plane of the wall and hence:

$$\mathbf{F} \cdot \mathbf{V} = 0$$

(II) The head of **F** is the shadow of the head of **P**. Consequently, the

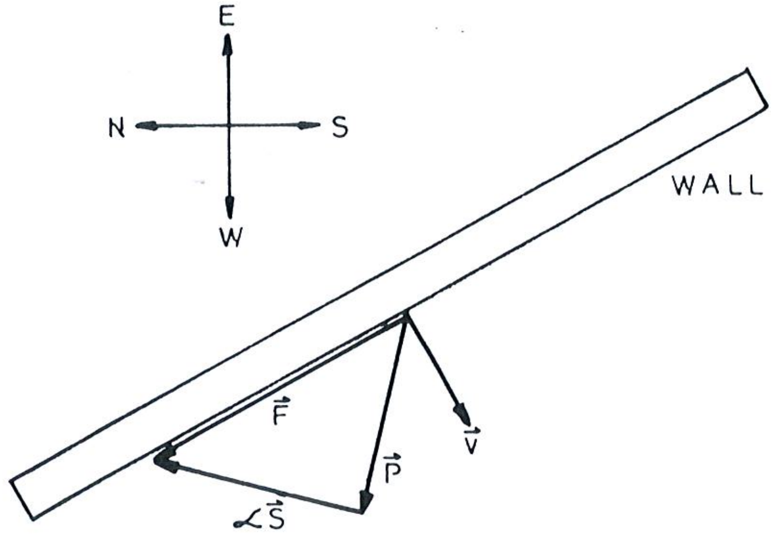

Fig. 4. *The geometry of the wall, the normal* **V** *and the pole* **P**.

heads of **F** and **P** must lie along the shadow vector **S**. We express it as:

$$\mathbf{F} = \mathbf{P} + \alpha\mathbf{S} \qquad (5)$$

where α is a scalar. The multiplication of the last equation by **V** and the application of condition (I) yields:

$$\alpha = -\mathbf{P}\cdot\mathbf{V}/\mathbf{S}\cdot\mathbf{V}$$

which ends the derivation of the shadow vector. The length of the shadow is given by the absolute value of **F**. The direction of the shadow on the wall is expressed by means of angle $\hat{F}$, the angle between the zenith and the shadow (see Fig. 5). $\hat{F}$ is given by:

$$\cos\hat{F} = \mathbf{F}\cdot\mathbf{Z}/|\mathbf{F}| \qquad (6)$$

There is ambiguity in the definition of $\hat{F}$ since the above formula does not distinguish between the two cases shown in Fig. 5A and 5B. We solve this problem by defining:

$$\hat{F} = \hat{F} \quad \text{for} \quad Fn > 0 \quad \text{(case 5A)}$$

and

$$\hat{F} = -\hat{F} \quad \text{for} \quad Fn < 0 \quad \text{(case 5B)}$$

where Fn is the north component of the shadow vector **F**.

The length of the shadow and its position (angle $\hat{F}$) on the wall are obtained from eqns. (5) to (7). The shadow vector **S** is found from eqn. (4). The normal **V**, which is determined by the azimuth of the wall as well as the component of the pole **P**, is input data for one calculation.

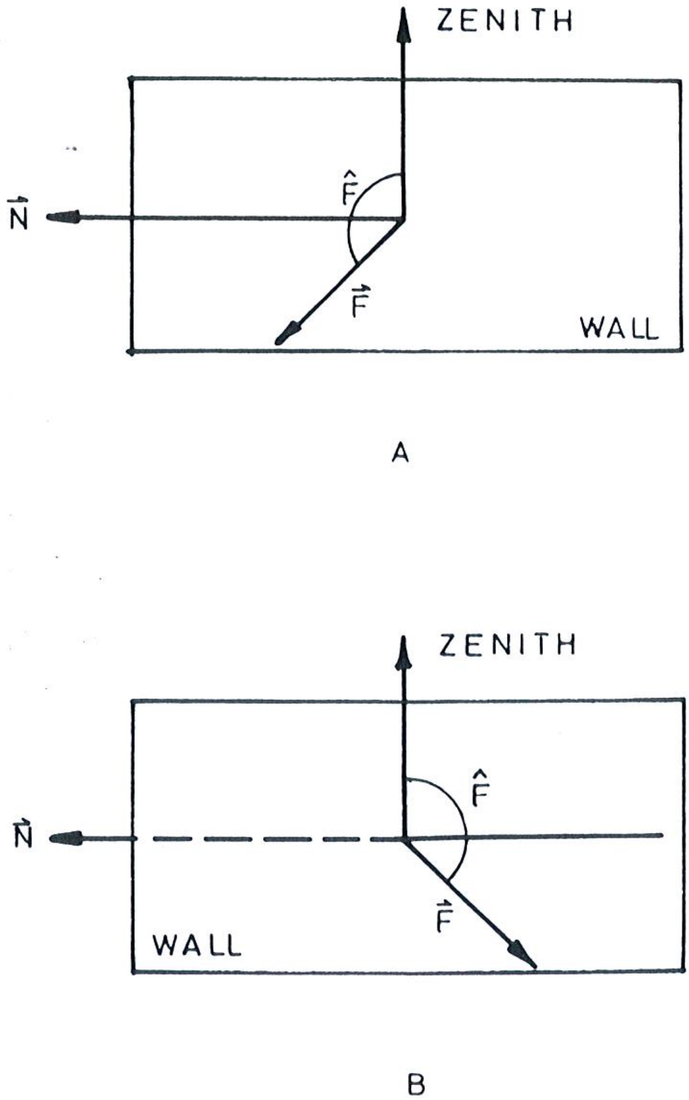

Fig. 5. (A) *The position of the shadow as seen in elevation.* (B) *The ambiguity in the definition of* $\hat{F}$.

We found it convenient to give **P** in terms of its length and the angles between **P** and the wall and between **P** and the horizon.

Calculation of sun-shades

Consider a window B cm high and A cm wide. The window is divided by a fine mesh (Fig. 6). The size of the mesh is arbitrary and is determined by the planner. In principle, a course mesh is sufficient for sun-shades made of few units, while a fine grid is needed for sun-shades made of many units or with special curvature. The variation of ZF, the angle between the shadow and the horizon (see Fig. 6) during the prescribed month and at the prescribed hours is calculated according to:

$$ZF = (|\hat{F}| - \pi/2) \text{ sign } SL \quad (8)$$

where SL is the horizontal component of the shadow, positive from left to right.

Considering the fine mesh shown in Fig. 6, we now imagine a pole of length l perpendicular to the wall at each mesh point. The length l is calculated in such a way as to give a shadow long enough to reach the frame of the window. Clearly, l depends on ZF, the geometry of the window and the mesh point under consideration. The shadow may reach the window-sill (point L, Fig. 6) or the frame (point S, Fig. 6). The length l is calculated from OL or OS, whichever is the case. This calculation is carried out for the 21st of every month for all the daytime hours the room is expected to be used. The distribution of l in the field of the window is the

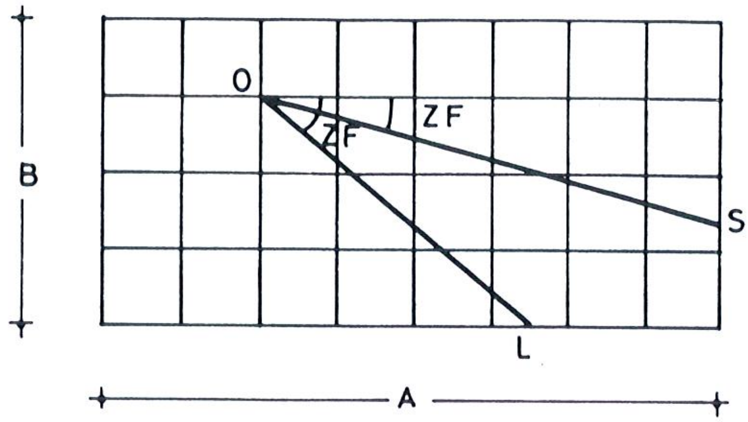

Fig. 6. *The division of the window to fine mesh. OS and OL are different orientations of the shadow cast by a pole at O.*

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MONTH= 6

6 21  8   NO SUN ON WALL
6 21  9   NO SUN ON WALL
M  D  H       ZF        SHAD-L    Z              A           PHI
SHADOW VECTOR PARALLEL TO WALL
6 21 10    -31.117      15.243     31.320    -82.765       3.753
    0.00     3.17     6.35      7.66      7.66      7.66      7.66      7.66      7.66
    0.00     3.17     5.75      5.75      5.75      5.75      5.75      5.75      5.75
    0.00     3.17     3.83      3.83      3.83      3.83      3.83      3.83      3.83
    0.00     1.92     1.92      1.92      1.92      1.92      1.92      1.92      1.92
    0.00     0.00     0.00      0.00      0.00      0.00      0.00      0.00      0.00
M  D  H       ZF        SHAD-L    Z              A           PHI
6 21 11    -17.693       8.103     18.999    -67.899       7.035
    0.00    10.15    12.95     12.95     12.95     12.95     12.95     12.95     12.95
    0.00     9.72     9.72      9.72      9.72      9.72      9.72      9.72      9.72
    0.00     6.48     6.48      6.48      6.48      6.48      6.48      6.48      6.48
    0.00     3.24     3.24      3.24      3.24      3.24      3.24      3.24      3.24
    0.00     0.00     0.00      0.00      0.00      0.00      0.00      0.00      0.00
M  D  H       ZF        SHAD-L    Z              A           PHI
6 21 12     -3.885       6.673      9.361    -24.332       8.523
    0.00    15.02    15.02     15.02     15.02     15.02     15.02     15.02     15.02
    0.00    11.27    11.27     11.27     11.27     11.27     11.27     11.27     11.27
    0.00     7.51     7.51      7.51      7.51      7.51      7.51      7.51      7.51
    0.00     3.76     3.76      3.76      3.76      3.76      3.76      3.76      3.76
    0.00     0.00     0.00      0.00      0.00      0.00      0.00      0.00      0.00
M  D  H       ZF        SHAD-L    Z              A           PHI
6 21 13     10.015       7.028     12.853     50.710       8.098
   14.45    14.45    14.45     14.45     14.45     14.45     14.45     14.45      0.00
   10.84    10.84    10.84     10.84     10.84     10.84     10.84     10.84      0.00
    7.22     7.22     7.22      7.22      7.22      7.22      7.22      7.22      0.00
    3.61     3.61     3.61      3.61      3.61      3.61      3.61      3.61      0.00
    0.00     0.00     0.00      0.00      0.00      0.00      0.00      0.00      0.00
M  D  H       ZF        SHAD-L    Z              A           PHI
6 21 14     23.682       9.853     24.341     75.819       5.795
   11.08    11.08    11.08     11.08     11.08     11.08     11.08      6.32      0.00
    8.31     8.31     8.31      8.31      8.31      8.31      8.31      6.32      0.00
    5.54     5.54     5.54      5.54      5.54      5.54      5.54      5.54      0.00
    2.77     2.77     2.77      2.77      2.77      2.77      2.77      2.77      0.00
    0.00     0.00     0.00      0.00      0.00      0.00      0.00      0.00      0.00
M  D  H       ZF        SHAD-L    Z              A           PHI
SHADOW VECTOR PARALLEL TO WALL
6 21 15     36.878      31.985     36.915     87.018       1.791
    3.91     3.91     3.91      3.91      3.91      3.91      2.60      1.30      0.00
    2.93     2.93     2.93      2.93      2.93      2.93      2.60      1.30      0.00
    1.95     1.95     1.95      1.95      1.95      1.95      1.95      1.30      0.00
     .98      .98      .98       .98       .98       .98       .98       .98      0.00
    0.00     0.00     0.00      0.00      0.00      0.00      0.00      0.00      0.00
6 21 16   NO SUN ON WALL
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Fig. 7. *Computer output of $l(x, y)$ for a southern window in June. The length of the pole, l, necessary to reach the frame of the window is given for every grid point x, y for all hours during which the sun shines on the wall. Time, according to Israel summer clock, is rounded to hours. Angles are given in degrees and lengths in cm.*

WEST=0.00 SOUTH=90.0 EAST=180.00 NORTH=270.00

MAXIMUM SHADES NEEDED IN MONTH 6 FOR WINDOW 200.00/100.00 IN WALL 90.00
A

0.00	15.02	15.02	15.02	15.02	15.02	15.02	15.02	15.02
12	12	12	12	12	12	12	12	12
0.00	11.27	11.27	11.27	11.27	11.27	11.27	11.27	11.27
12	12	12	12	12	12	12	12	12
0.00	7.51	7.51	7.51	7.51	7.51	7.51	7.51	7.51
12	12	12	12	12	12	12	12	12
0.00	3.76	3.76	3.76	3.76	3.76	3.76	3.76	3.76
12	12	12	12	12	12	12	12	12
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0	0	0	0	0	0	0	0

MAXIMUM SHADES NEEDED IN MONTH 6 FOR WINDOW 200.00/100.00 IN WALL 90.00
B

14.45	14.45	14.45	14.45	14.45	14.45	14.45	14.45	0.00
13	13	13	13	13	13	13	13	14
10.84	10.84	10.84	10.84	10.84	10.84	10.84	10.84	0.00
13	13	13	13	13	13	13	13	14
7.22	7.22	7.22	7.22	7.22	7.22	7.22	7.22	0.00
13	13	13	13	13	13	13	13	14
3.61	3.61	3.61	3.61	3.61	3.61	3.61	3.61	0.00
13	13	13	13	13	13	13	13	14
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0	0	0	0	0	0	0	0

Fig. 8. *Computer output of l_{max}(day) as a function of x, y for a southern window in June. A is for the sun shining from the east and B for the sun shining from the west.*

fundamental information for the design of proper sun-shades. Typical results are shown in Fig. 7. Here we have a window on a southern wall, in the northern hemisphere (latitude N + 32°7′ and longitude 34°47′ East of Greenwich). The calculation is carried out for the 21st of June from 8 am to 4 pm. The mesh contains 9 × 5 points at equal spacing. In the example the width to height ratio for the window is 200 cm/100 cm. The symbols in Fig. 7 have their usual meaning except for *SHAD-L* which is the length of shadow cast by a pole of standard length (100 cm) and *PHI*—the angle between the shadow vector and the wall. This angle may be needed under certain circumstances.

The next step is to find the maximum of l over the day for every month at each mesh point (see Fig. 8). This maximum is denoted by l_{max}(day). Here we distinguish between two cases: the solar rays come from right to left (Fig. 8A) or left to right (Fig. 8B). Figure 8 shows the maximum of l for each mesh point, as found from Fig. 7, for a southern window in June. Figure 8 also provides the hour at which the maximum was reached at each mesh point. Note that results in Figs. 8A and 8B do not refer to a maximum at a given moment. The maxima at distinct grid points are sometimes reached at different times.

The numerical results for l_{max}(day) are presented graphically by means of an axonometric projection which is plotted for every month. Figure 9 summarises the results for a southern window for all months. These drawings simplify the design of sun-shades according to prescribed conditions. By way of illustration note that, in winter, when the sun does not rise high above the horizon, the necessary dimensions of the shades are very large. However, the penetration of solar radiation may be desired in winter, since the little heating it causes is comfortable. (This is, of course, true to the extent that the heating system is flexible and can take this fact into account by thermostats in some southern rooms, for example.) When this is the case the design should be such as to prevent direct solar radiation in summer only.

The final step is the calculation of l_{max}(month) the maximum of l_{max}(day) over all months during which direct sunlight is not desired. We again distinguish between two cases, depending on the direction of the shadow. Typical results, which refer to the southern window discussed above, are given in Fig. 10. Figure 10A corresponds to the sun shining from right to left and Fig. 10B for the opposite direction. The month and hour at which l_{max}(month) is obtained is given below the numerals. The above separation into

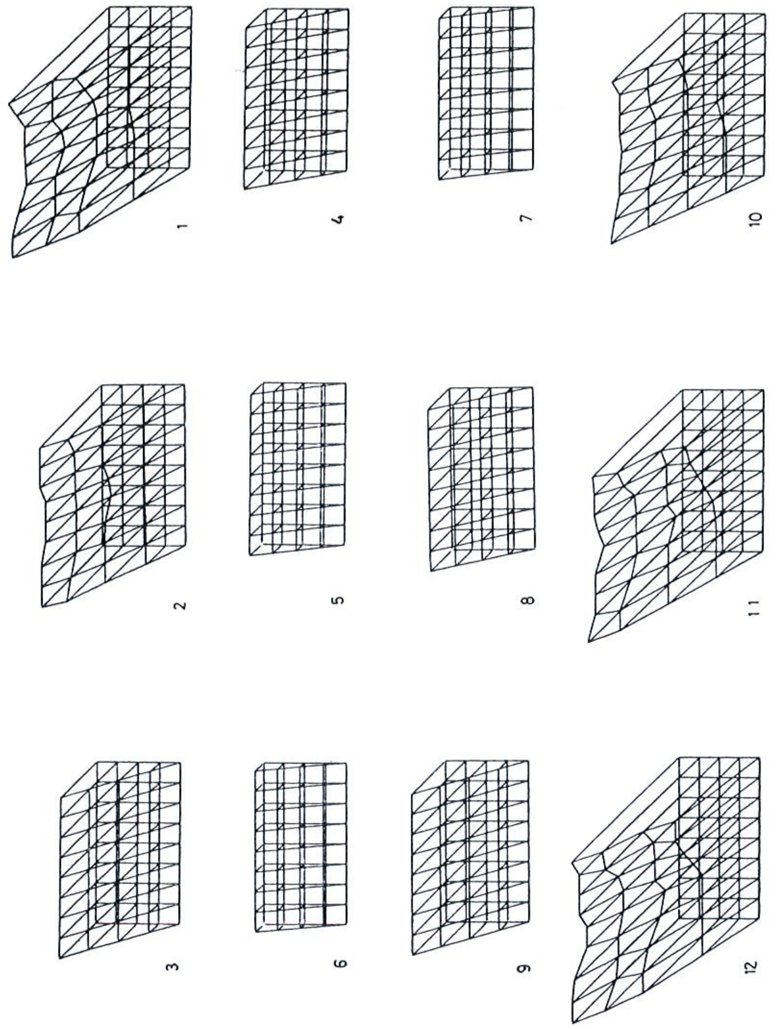


Fig. 9. *An axonometric projection of* I_{max}*(day) for a southern window for all months.*

WEST=0.00 SOUTH=90.0 EAST=180.00 NORTH=270.00

MAXIMUM SHADES NEEDED FOR WINDOW 200.00/100.00 IN WALL 90.00

A

0.00	96.95	96.95	100.17	100.17	108.07	108.07	108.07	122.18
10 12	10 12	10 12	10 11	10 11	10 10	10 10	10 10	10 9
0.00	72.71	75.13	75.13	81.05	81.05	91.63	95.13	95.13
10 12	10 12	10 11	10 11	10 10	10 10	10 9	10 9	10 9
0.00	48.47	50.09	54.04	61.09	63.42	63.42	69.75	79.71
10 12	10 12	10 11	10 10	10 9	10 9	10 9	10 8	10 8
0.00	25.04	30.54	31.71	39.86	47.78	47.78	47.78	47.78
10 12	10 11	10 9	10 9	10 8	10 8	10 8	10 8	10 8
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10 0	10 0	10 0	10 0	10 0	10 0	10 0	10 0	10 0

MAXIMUM SHADES NEEDED FOR WINDOW 200.00/100.00 IN WALL 90.00

B

122.19	114.92	105.22	106.22	102.83	99.36	96.71	96.71	0.00
10 16	10 16	10 15	10 15	10 15	10 14	10 13	10 13	10 16
91.64	91.64	91.64	82.09	79.66	77.12	74.52	72.53	0.00
10 16	10 15	10 16	10 16	10 15	10 15	10 14	10 13	10 16
61.09	61.09	61.09	61.09	61.09	53.11	51.41	48.36	0.00
10 16	10 16	10 16	10 16	10 16	10 15	10 15	10 13	10 16
30.55	30.55	31.55	30.55	30.55	30.55	30.55	25.71	0.00
10 16	10 16	10 16	10 16	10 16	10 16	10 16	10 15	10 16
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10 0	10 0	10 0	10 0	10 0	10 0	10 0	10 0	10 0

Fig. 10. *Computer output of l_{max}(month) as a function of x, y for a southern window over the period March 21st to October 21st and from 8 am to 4 pm Israel summer time. A and B are for the sun shining from the east and west respectively.*

two cases is irrelevant for the design of horizontal sun-shades, but this is not the case with vertical shades. Here the direction of the sun is important. In the case of both vertical and horizontal shades, the size depends on the shades' structure. If a single-unit sun-shade is designed, the size l_{max}(month) must be found for mesh points along the lintel for horizontal shades (Fig. 11A), or along the frame of the window on the side from which the sun shines, for vertical shades (Fig. 11B). Sometimes n-unit shades are desired. In that case the size of the shades should be l_{max}(month) found for mesh points a distance B/n from the windowsill for horizontal shades (Fig. 11A), or l_{max}(month) found for mesh points a distance A/n from the frame of the window on the opposite side to the direction from which the sun shines for vertical shades (Fig. 11B). It is important to note that Fig. 11A for horizontal, and Fig. 11B for vertical, shades are not identical. The dependence of l_{max}(month) on the location in the window is demonstrated in an isometric projection (Fig. 11). Such a drawing helps in the conceptual process of the design. The actual measures are easily found from the figures.

The solution for solar-shades is by no means unique. One can design efficient sun-shades of different shapes that may render the external appearance of a building very attractive. Of course, the design depends on the dimensions and form of the window, as well as on climatological conditions and functional demands. The approach presented here should be contrasted with the traditional one,[6,7] in which a sun-shade of a given form is assumed first and then a check on its performance, and the extent to which it complies with the architectural demands, is made. Here the logical sequence is inverted. At first, a calculation of the maximum length, l, of a pole (perpendicular to the plane of the window and located at a general point in the window) needed to cast a sufficiently long shadow is carried out. The design starts only in the second step. One obtains in this way good, efficient, and attractive sun-shades.

Sun-shades in an office building—an example

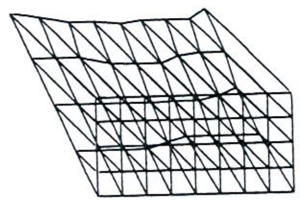
In this section it is intended to demonstrate the method as applied to the design of sun-shades for an office building. The location is Tel Aviv, Israel (latitude N + 32°7′ and longitude 34°47′ East of Greenwich).

Tel Aviv is located along the mediterranean coast. The climate is hot—humid in summer and pleasant in winter. The average number of rainy days is about 30. Due to the above considerations we look for sun-shades that prevent direct sun

A

MAXIMUM SHADES NEEDED FOR WINDOW 200.00/100.00 IN WALL 90.00

122.19 10 16	114.92 10 16	106.22 10 15	106.22 10 15	102.83 10 15	108.07 10 10	108.07 10 10	108.07 10 10	122.18 10 9
91.64 10 16	91.64 10 16	91.64 10 16	82.09 10 16	81.05 10 10	81.05 10 10	91.63 10 9	95.13 10 9	95.13 10 9
61.09 10 16	61.09 10 16	61.09 10 16	61.09 10 16	61.09 10 16	63.42 10 9	63.42 10 9	69.75 10 8	79.71 10 8
30.55 10 16	30.55 10 16	30.55 10 16	31.71 10 9	39.86 10 8	47.78 10 8	47.78 10 8	47.78 10 8	47.76 10 8
0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0



B

MAXIMUM SPACES NEEDED FOR WINDOW 200.00/100.00 IN WALL 90.00

0.00 10 12	96.95 10 12	96.35 10 12	100.17 10 11	102.83 10 15	108.07 10 10	108.07 10 10	114.92 10 16	122.19 10 16
0.00 10 12	72.71 10 12	75.13 10 11	77.12 10 15	81.05 10 10	82.09 10 15	91.64 10 16	95.13 10 9	95.13 10 9
0.00 10 12	48.47 10 12	51.41 10 15	54.04 10 10	61.09 10 16	63.42 10 9	63.42 10 9	69.75 10 8	79.71 10 8
0.00 10 12	25.71 10 15	30.55 10 16	31.71 10 9	39.86 10 8	47.78 10 8	47.78 10 8	47.78 10 8	47.78 10 8
0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0	0.00 10 0

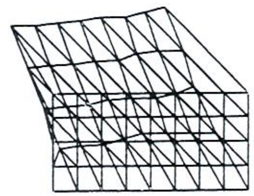


Fig. 11. *Computer output of l_{max}(month) for a southern window and the period and hours considered. A shows the maximum of l_{max}(month) over the two possible directions of the solar radiation; B shows maximum depth needed for vertical shades.*

from March 21 until October 21. Typical office hours are 8 am to 4 pm. The building is rectangular and the windows are in the south, west, east and north walls.

The first run was made without any seasonal constraints, *i.e.* l_{max}-(day) was found for the 21st of each month. A glance at the accompanying figures shows that very large shades are needed in winter for the southern, eastern and western windows. The phenomenon is most prominent in a southern window (Fig. 9). At this latitude, a northern window will get direct sun only in summer (and even then only in the early morning or late afternoon). These facts and considerations led us to demand no penetration of direct sun from March 21st until October 21st. This demand implies that penetration of direct sun in winter is possible. Note that l_{max}-(month) is a function of x and y where x and y are the coordinates of the mesh point. Hence the maximum at different points, x and y, may be reached in different months and at different hours. This fact is most pronounced in a southern window. Since the most effective shape of the sun-shade is not known *a priori*, it is necessary to search for l_{max}(month) during the whole relevant season and at all relevant hours.

The results obtained for the different windows are dissimilar. A design of a general sun-shade, equally effective for all window orientations, will give rise to an inefficient shape. One can, in principle, search for a sun-shade that prevents the penetration of direct sun through all windows in the building. This is easily accomplished by looking for the maximum of l_{max}(month) over all relevant orientations. However, the properties of maximum natural illumination and good cover from direct sun over an extended period are opposed to each other. If the sun-shade is very big it will not allow abundant natural illumination, and vice-versa. This explains why we find that such a solution fails to maximise the natural illumination by scattered and reflected light and to allow the penetration of the desired winter sun. In such a solution the sun-shade for a given window is designed to account for situations that occur in other windows but may never happen in this particular one. We decided, therefore, in this particular case of a north, east, south and west oriented building to separately design sun-shades for each of the four orientations of the building.

A southern window

Results for a southern window are summarised in Fig. 11A and B. A glance at Fig. 11B for vertical sun-shades shows that such shades are ineffective for this direction, since the size of the shade does not depend on the width of the window. Hence, if one or several vertical shades are designed, their (big) size must always remain the same. Moreover, a check of the azimuth A (Fig. 1) of the sun, shows that it varies from $-90°$ to $+90°$, during those hours in which direct solar rays fall on the southern wall. Hence it is impossible to have only vertical shades tilted in any angle to the wall and to prevent the penetration of direct sun. It follows that a solar-shade for a southern window must be basically horizontal. It need not be parallel to the horizon, but its horizontal projection must be large.

Several design alternatives for a southern window are shown in Fig. 12. Alternatives 12B–D show some possibilities of horizontal units with small vertical elements on the sides. Alternative 12B shows a single horizontal unit on the lintel. The depth of this unit must be 122 cm when measured from the inner wall of the building, that is one has to subtract the wall thickness from 122 cm. All measures in the figures refer to distance (in cm) from the inner wall of the building. Alternative 12C demonstrates the possibility of using two identical horizontal units, each sheltering half a window. The measures of the shades decrease to 80 cm on one side and 61 cm on the other. Alternative 12D shows a combination of four small sun-shades placed at equal distances of 25 cm apart. The depth of the shade decreases to 48 cm on one side and 31 cm on the other.

A solution of a different type is shown in Fig. 12E. Here the design is based on the conditions at $x = 0{\cdot}75B$ (three-quarters of the height of the window). When the depth of the shade is taken as 95 cm it solves the problem of the lower part of the window. To solve the problem of the upper part, a tilted awning is suggested. When the tilt is increased, the depth can be decreased to 80 cm, as shown in Fig. 12G.

Another proposition (Fig. 12F) is to leave a horizontal element parallel to the plane of the window (but at a distance of 80 cm away). The vertical element should reach half the height of the window because 80 cm corresponds to l_{max}(month) at $x = 0{\cdot}5B$ and $y = A$ (the edge of the window).

The particular behaviour of l_{max}-(month) for $x = 0{\cdot}5B$ and different y and, in particular, the rapid increase of l_{max}(month), $x = 0{\cdot}5B$ for y close to A imply the feasibility of vertical side elements inclined towards the window plane at an acute angle. An example is shown in Fig. 12H.

Figures 12I and 12J demonstrate the space of possibilities when the window is divided into several smaller parts. Thus, alternative 12I is derived from 12H when the

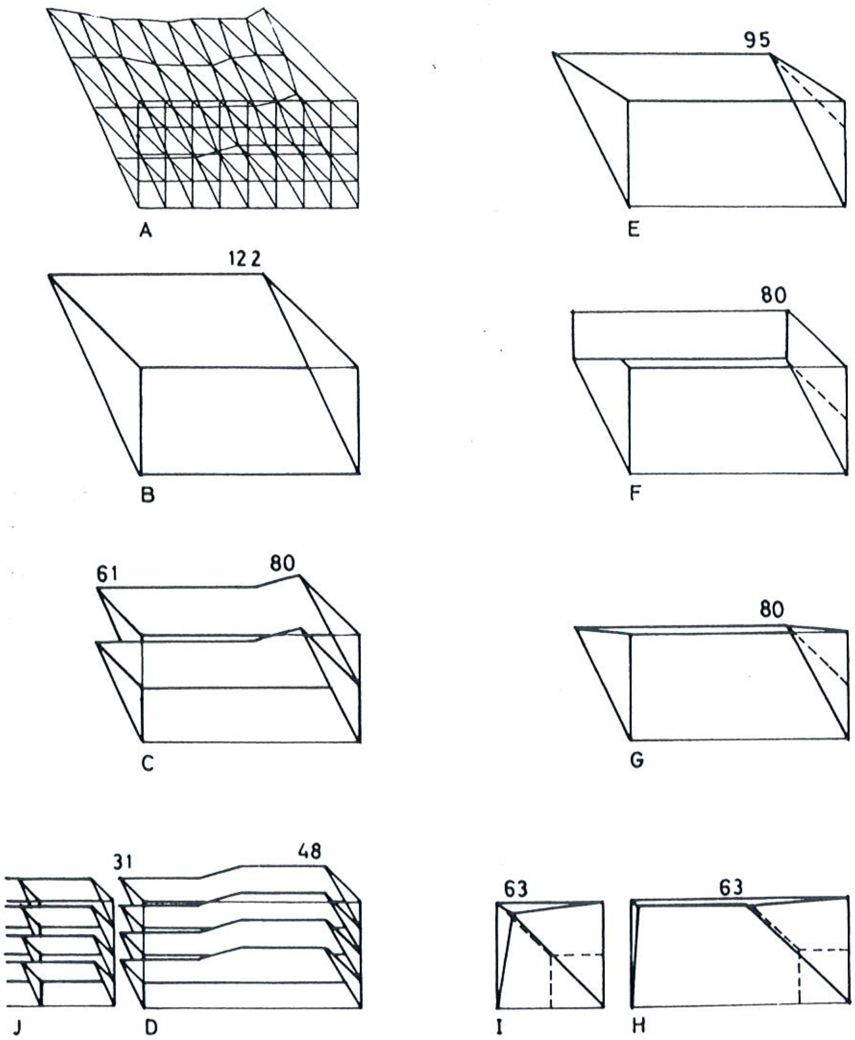


Fig. 12. *An axonometric projection of l_{max}(month) for a southern window and shading in the period and at the hours considered (A). B–J alternatives for sun-shades.*

200 × 100 cm window is considered as two adjacent windows, each 100 × 100 cm. Similarly 12J is derived from 12D.

In all the examples shown in Fig. 12 vertical elements are added on the sides of the cantilever part. These vertical elements can sometimes be cancelled and compensated for by prolonging the cantilever beyond the window. The extension is easily calculated from $\Phi = PH1$, the angle between the solar rays and the wall (see Fig. 7). A sun-shade with extended cantilever allows the penetration of direct winter sun for more hours than one with vertical elements.

The examples shown in Fig. 12 do not exhaust the possibilities. All examples comply with the fundamental demand: no penetration of direct sunlight during the above period and at the above hours.

A western window

The constraints on a sun-shade for a western window are no penetration of direct sunlight during the period March 21st to October 21st and until 4 pm (the morning hour constraint has, of course, no meaning in this case). The results for l_{max}(month) are shown graphically in Fig. 13A. The depth of the necessary shade is very large and the solution quite expensive. We suggest, therefore, permitting the penetration of direct sunlight during the summer after 3.30 pm. In Fig. 13B and Fig. 14 we show the results obtained when the time of shading in summer is shortened by half-an-hour. This compromise is not a serious drawback on the energy balance of the room. The penetration of relatively low summer sunlight will heat the room during the last half-hour of the working day. This time is too short for the room to reach uncomfortable temperatures. On the other hand, the compromise shown in Fig. 13B allows the penetration of winter sunlight from November 21st until January 21st after 2 pm and in spring and autumn after 3 pm.

The question of vertical shades deserves special attention in this case. The azimuth of the sun varies from 0° to +90° during the early afternoons of the summer (earlier than 3.30 pm). The sun always shines from right to left or perpendicular to the wall. Hence the tables for horizontal and vertical shades are identical (Fig. 14). The results are shown graphically in Fig. 13B. It follows that vertical sun-shades are feasible. An example is shown in Fig. 13K, where a plan (horizontal sections) of vertical shades inclined at 45° to the wall is drafted. Note that a sun-shade made of only vertical elements is not sufficient to satisfy the constraints. It is necessary to add small horizontal elements or to extend the

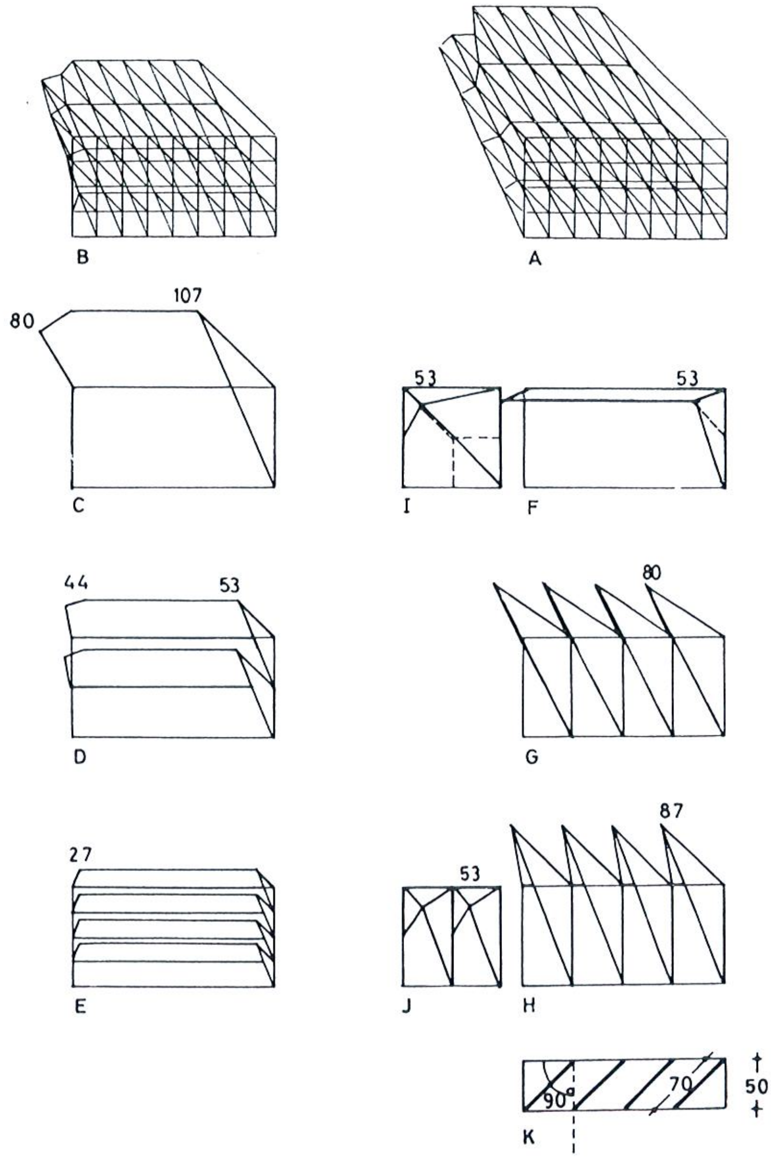


Fig. 13. *An axonometric projection of* l_{max}*(month) and some alternative designs of sun-shades for a western window.*

WEST=0.00 SOUTH=90.0 EAST=180.00 NORTH=270.00

MAXIMUM SHADES NEEDED FOR WINDOW 200.00/100.00 IN WALL 0.00

0.00	79.91	87.54	106.91	106.91	106.91	106.91	106.91	106.91
10 15	7 15	3 15	3 15	3 15	3 15	3 15	3 15	3 15
0.00	65.65	80.18	80.18	80.18	80.18	80.18	80.18	80.18
10 15	8 15	3 15	3 15	3 15	3 15	3 15	3 15	3 15
0.00	43.77	53.45	53.45	53.45	53.45	53.45	53.45	53.45
10 15	8 15	3 15	3 15	3 15	3 15	3 15	3 15	3 15
0.00	26.73	26.73	26.73	26.73	26.73	26.73	26.73	26.73
10 15	3 15	3 15	3 15	3 15	3 15	3 15	3 15	3 15
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10 0	10 0	10 0	10 0	10 0	10 0	10 0	10 0	10 0

Fig. 14. *Computer output of l_{max}(month) for a western window.*

vertical shade beyond the lintel. Such a solution completely blocks the view in the western direction. This is an undesirable feature since (a) no winter sunlight will ever penetrate through the window and (b) it causes a great reduction in natural illumination.

It is a common misconception on the part of architects that the sun-shades of a western window must be vertical. Our results show that for the period and at the hours considered here the depth of a horizontal shade is 107 cm (Fig. 13C) in the case of a single element, or 53 cm in the case of two elements (Fig. 13D), etc. These measurements are smaller than the corresponding ones in a southern window. Hence, horizontal sun-shades are obviously practical for this western window. Moreover, contrary to the case of a southern window, the vertical element is required only on the right side of the window. The same conclusion applies to the extension of the horizontal element, or the tilted horizontal awning shown in Fig. 13F.

A comparison of horizontal sun-shades (Fig. 13D) with the vertical sun-shade (Fig. 13H) is instructive. While the depth of the horizontal shade is only 53 cm, the depth of the vertical shade is 87 cm. Even when the shades are tilted to the wall (Fig. 13G) the depth is still 80 cm. Consequently, vertical shades need more material to produce the same shading as horizontal shades. On the other hand, a comparison of the two solutions of vertical shades (Fig. 13G and 13K) shows that natural illumination is greater in 13G, and there is no blocking of the west.

Similar conclusions about the superiority of horizontal over vertical shades were derived by Givoni,[6] who looked for the performance of a given sun-shade. The results presented here show that the conclusion is independent of the shape of the horizontal or vertical shades, but may depend on the period and the hours during which shading is required, as well as on the latitude. The possibilities of dividing the window into two or more small adjacent windows are demonstrated in Figs. 13J and 13E respectively. The penetration of low winter sunlight is permitted in these examples.

An eastern window

The requirements for shading an eastern window are the period of March 21st to October 21st and after 8 am. The results for l_{max}-(month) are shown in Fig. 15A. The situation here resembles that found in a western window, *i.e.* very large shades are needed. Hence, we decided to relax the constraint and shorten the time span to no direct sunlight from 9 am on, but to keep

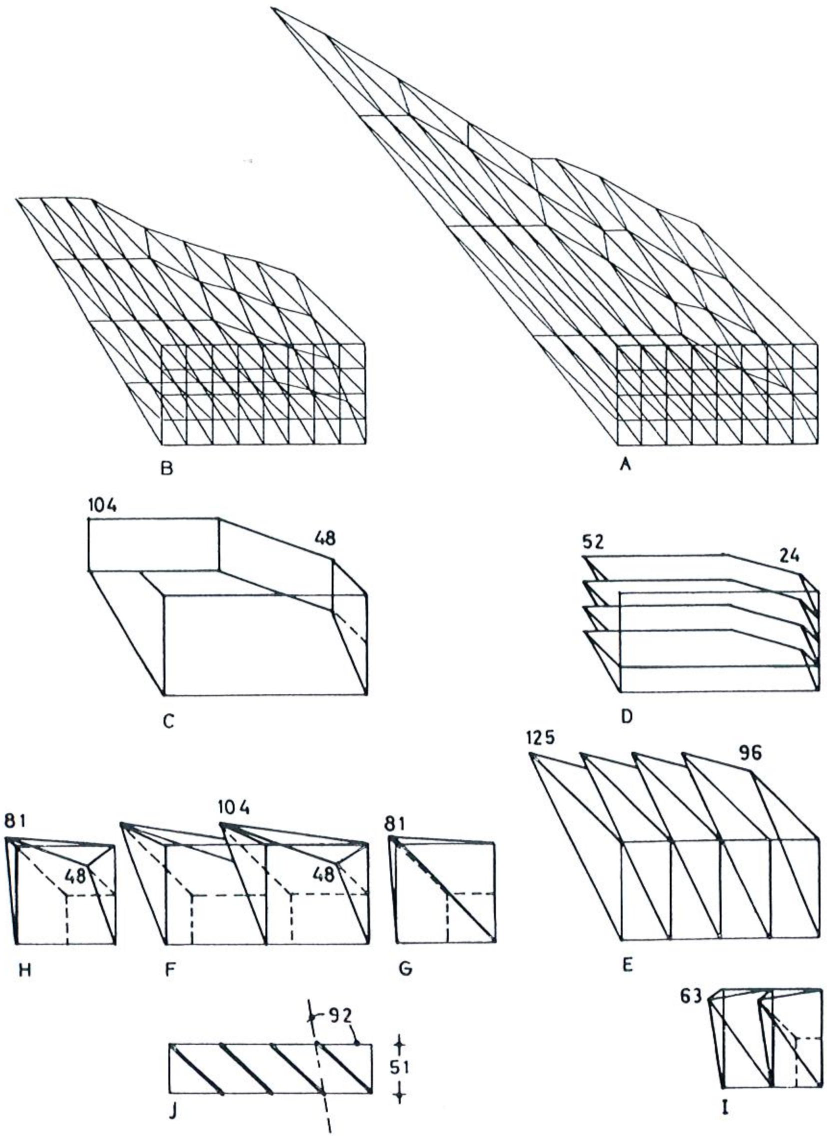


Fig. 15. *An axonometric projection of* l_{max}*(month) and some alternative designs of sun-shades for an eastern window.*

the period constraint. The results with this relaxed constraint are shown in Fig. 15B. The depth of the shades is smaller and hence they allow penetration of direct sunlight in winter until 10 am. The general problems of an eastern window resemble those of a western one. The azimuth $\hat{A}$ of the sun varies from $-92°$ to $0°$ during the relevant hours. The solar rays generally come from left to right and, for a short while, from right to left. This fact gives rise to the identity of the tables for horizontal and vertical shades (Fig. 16). It is possible to design vertical sun-shades which are inclined to the wall. The inclination angle, depth and number of units should be such as to prevent penetration of sunlight in the azimuth range $-92°$ to $0°$. An example is shown in Fig. 15J, for an inclination angle of 45°. The view to the east is completely blocked in this case.

Contrary to the case of a western window, it is unpractical to have a single horizontal unit, since its depth must be 208 cm. A better solution is four horizontal elements (Fig. 15D) about 52 cm deep. Note that the depth to window ratio is about 2:1 in all cases.

Vertical shades must also be very deep. A vertical sun-shade made of four units 50 cm apart must be 125 cm deep (Fig. 15E). The depth decreases to only 116 cm when eight vertical elements are used. Figure 15C shows a special combination of horizontal and vertical elements. A symmetrical treatment of both sides in examples 12C, D and E (*i.e.* assuming a constant depth of 104 cm in 12C, etc.) results in a sun-shade capable of preventing the penetration of direct sunlight during spring and summer months (April to August) already at 8 am. This may be a preferred solution.

It is basically the large width of the window (200 cm) which gives rise to such big sun-shades. The division of the window into smaller units and the use of tilted sun-shades may improve the situation (Fig. 15G, F, and H). The symmetrical solutions 15G and I, that is the cases in which the depth of the sun-shade is the same on the right and on the left sides, prevent the penetration of direct solar rays in the period April to August at 8 am. This is done at the cost of reducing natural illumination during the day.

We find again that the common architectural practice of using vertical shades on the eastern side is wrong. While the efficiency of a horizontal shade is greater than that of a vertical one (in terms of area of shading device versus area shaded), it is apparently better to divide the window into smaller ones. The importance of adjusting the dimensions of the window to the orientation is clear. We emphasise that the

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WEST=0.00   SOUTH=30.0   EAST=180.00   NORTH=270.00

MAXIMUM SHADES NEEDED FOR WINDOW 200.00/100.00 IN WALL 180.00
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207.64	207.64	207.64	204.62	163.70	144.89	125.41	115.84	96.59
10 9	10 9	10 9	10 9	10 9	9 9	9 9	4 9	6 9
155.73	155.73	155.73	155.73	155.73	122.77	108.67	86.88	72.44
10 9	10 9	10 9	10 9	10 9	10 9	9 9	4 9	6 9
103.82	103.82	103.82	103.82	103.82	103.82	81.85	62.70	48.29
10 9	10 9	10 9	10 9	10 9	10 9	10 9	9 9	6 9
51.91	51.91	51.91	51.91	51.91	51.91	51.91	40.92	24.15
10 9	10 9	10 9	10 9	10 9	10 9	10 9	10 9	6 9
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10 0	10 0	10 0	10 0	10 0	10 0	10 0	10 0	10 0

Fig. 16. *Computer output of l_{max}(month) for an eastern window.*

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WEST=0.00   SOUTH=90.0   EAST=180.00   NORTH=270.00

MAXIMUM SHADES NEEDED FOR WINDOW 200.00/100.00 IN WALL 270.00


22.77   22.77   22.71   18.92   15.14   11.35    9.81    9.81    9.81
 6  8    6  8    6  8    6  8    6  8    6  8    6 16    6 16    6 16

17.07   17.07   17.07   17.07   15.14   11.35    7.57    7.36    7.36
 6  8    6  8    6  8    6  8    6  8    6  8    6  8    6 16    6 16

11.38   11.38   11.38   11.38   11.38   11.35    7.57    4.91    4.91
 6  8    6  8    6  8    6  8    6  8    6  8    6  8    6 16    6 16

 5.69    5.69    5.69    5.69    5.69    5.69    5.69    3.78    2.45
 6  8    6  8    6  8    6  8    6  8    6  8    6  8    6  8    6 16

 0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00    0.00
10  0   10  0   10  0   10  0   10  0   10  0   10  0   10  0   10  0
```

Fig. 17. *Computer output of l_{max}(month) for a northern window.*

above conclusion depends on the required period and hours of shading.

A northern window

The numerical results for a northern window are given in Figs. 17 and 18. We find that l_{max}(month) reaches its maximal value of 23 cm at 8 am in June. Typical concrete walls are about this thickness and hence no shades are needed. In the case of larger windows, a small shade is sufficient.

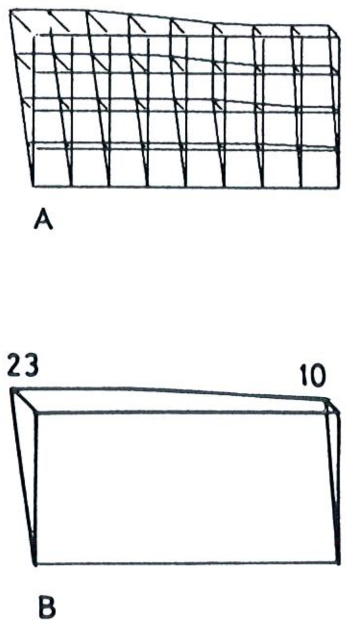

Fig. 18. *An axonometric projection of l_{max}(month) and a possible sun-shade for a northern window.*

Summary and conclusions

The steps of the method are:

1. The azimuth $\hat{A}$ and the zenith angle $\hat{Z}$ of the sun are found for the period and hours during which shading is required.
2. The window is divided into a fine mesh. A perpendicular pole is imagined at every grid point and the length necessary to cast a shadow to the window frame is calculated for all grid points.
3. The maximum length of the necessary pole, l_{max}, at each point is calculated over the days and months in the relevant period.
4. The results for l_{max}(month) are presented graphically using an axonometric projection.

The latitude and longitude of the building and the orientation and dimensions of the windows, are all input data for the model. The efficiency of this procedure should be compared with the tedious traditional one in which a horizontal, vertical or tilted shading device is assumed, the path of the sun is calculated from nomograms and the performance of the shade is found. The numerical results and graphical presentation obtained with this method enable the designer to consider all possibilities at once and obtain an efficient sun-shade that satisfies the shading requirements. Moreover, the design of efficient non-standard shades with many variations, and the enrichment of the shapes of the elevations of buildings, is possible.

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References

1. HOPKINSON, R. G. and KAY, J. D., *The lighting of buildings*, Faber and Faber Pub. Comp., 1972.
2. LANDOLT-BORNSTEIN. Numerical data. Group VI. Volume 1: *Astronomy and astrophysics*, Springer-Verlag, Berlin, 1965.
3. LYNES, J. A., *Principles of natural lighting*, Applied Science Publishers Ltd, London, 1968.
4. GIVONI, B., NEEMAN, E., HOFFMAN, M. E. and BECKER, M., Effect of orientation and details of windows and fixed shading devices upon indoor temperature and illumination conditions of classrooms, Building Research Station. Technion, Israel Institute of Technology, Haifa, 1968.
5. ANON., *The American Ephemeris and Nautical Almanac for the year* 1973, US Government Printing Office, Washington, 1973.
6. GIVONI, B., *Man, climate and architecture*, Applied Science Publishers Ltd, London, 1969.
7. ANON., Application of Climatic Data to House Design, Housing and Home Finance Agency, Washington, D.C., 1954.

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